1. Interpret the physical meanings of Fourier series and Fourier transform, respectively. (10%)

2. Solve the following partial differential equation and discuss the corresponding eigenvalues and eigenfunctions:

\[ \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - G \]

(G: the acceleration of gravity)

with boundary conditions: \( u(0, t) = 0 \), \( u(L, t) = 0 \);

and initial conditions: \( u(x, 0) = f(x) \) and \( u_t(x, 0) = 0 \).

Also demonstrate the physical meanings of eigenvalue and eigenfunction for this equation. (25%)

3. Evaluate

\[ \int_{-\infty}^{\infty} \frac{dx}{(x-1)(x^2+3)} \]

(10%)

4. Solve the following equations.

(a) \( y'' + 2y' + 2y = 0 \), \( y(0) = -y'(0) = 1 \).

(b) \( y(t) = t + \int_0^t y(\alpha) \sin(t-\alpha) d\alpha \). (20%)

5. Know that a matrix \( A \), then to

(a) evaluate the inverse (if it exits), and (b) find a basis of eigenvectors and diagonalize. (20%)

\[ A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix} \]

6. (a) Describe the divergence theorem of Gauss.

(b) Evaluate \( I = \iint_S 3xydz - x^2ydx + x^2zdxdy \), where \( S \) is the surface of \( x^2 + y^2 + z^2 = 1 \). (15%)