1. The apparatus shown in Fig. 1 is used to measure the air velocity at the center of a duct having a 10-cm diameter. A tube mounted at the center of the duct has a 2-mm diameter and is attached to one leg of a slant-tube manometer. A pressure tap in the wall of the duct is connected to the other end of the slant-tube manometer. The well of the slant-tube manometer is sufficiently large that the elevation of the fluid in it does not change significantly when the fluid moves up the leg on the manometer. The air in the duct is at a temperature of 20°C, and the pressure is 150 kPa. The manometer liquid has a specific gravity of 0.8, and the slope of the leg is 30°. When there is no flow in the duct, the liquid surface in the manometer lies at 2.3 cm on the slanted scale. When there is flow in the duct, the liquid moves up to 6.7 cm on the slanted scale. Assuming a uniform velocity profile in the duct, calculate the flow rate of the air. Find the velocity of the air in the duct. (25%) 

![Fig. 1](image1.png)

2. As shown in Fig. 2, a windmill is operating in a 10 m/sec wind that has a density of 1.2 kg/m³. The diameter of the windmill is 4 m. The constant pressure (atmospheric) streamline has a diameter of 3 m upstream of windmill and 4.5 m downstream. Assume the velocity distributions are uniform and the air is incompressible; determine the thrust on the windmill. (25%) 

![Fig. 2](image2.png)
3. If the variables of interest for the pump applications are the head per weight, \( H \), the power \( P \), the diameter, \( D \), the flow rate, \( Q \), the rotation speed, \( N \) and viscosity, \( \mu \), and the density \( \rho \). Namely, \( g(D,\mu,\rho,N,H,Q,P)=0 \). (a) Select the variables \( \rho, N, D \) as the repeating parameters, and using the Bingham \( \pi \) theorem to find the dimensionless parameters groups; (10%) (b) A centrifugal pump has an efficiency of 80\% at its design point specific speed of 2300 (unit of rpm, \( m^3/hr \), and m). The impeller diameter is 200 mm. At design-point flow conditions, the volume flow rate is 68 \( m^3/hr \) of water at 1170 rpm. To obtain a higher flow rate, the pump is to be fitted with an 1750 rpm motor. Use the similarity laws for pumps applications to find the actual power input (\( P \)) required for this application, head (\( H \)) at higher flow rate condition. (10%)

\[
N_S = \left( \frac{\rho Q^2}{H} \right)^{3/4}
\]

4. Consider a non-Newtonian fluid between two parallel plate separated by a distance \( 2h \), the velocity profile for the fully-developed laminar flow of this fluid is given by

\[
 u(y) = \left( \frac{\Delta p}{\mu L} \right)^{1/n} \left[ \frac{nh}{n+1} \right] \left[ 1 - \left( \frac{y}{h} \right)^{(n+1)/n} \right]
\]

where \( y \) is measured from the centerline between these two plates, \( \mu \) is the viscosity of the fluid, \( L \) is the length of the plates, \( \Delta p \) represents the pressure difference between the inlet and the outlet, and \( n \) is constant. Prove that the flow rate for this flow may be expressed by

\[
 Q = \left[ \frac{h \Delta p}{\mu L} \right]^{1/n} \cdot \frac{2nWh^2}{2n+1}
\]

\( W \) is the width of the plates. (10%)

5. A depicted in Figure 5, it has been proposed to use the oil drums to make simple windmills for underdeveloped countries. Two possible configurations are shown. Estimate which (the right or the left one) would be better (10%)? How much is the difference between the two configurations. (10%)

Figure 5
Some useful equations:

- The continuity equations:
  \[ \frac{1}{r} \frac{\partial}{\partial r} \left[ ru_r \right] + \frac{\partial}{\partial \theta} [u_\theta] + \frac{\partial}{\partial z} [u_z] = 0 \quad \text{Or} \quad \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0 \]

- The Navier-Stokes (or momentum) equations in polar coordinate are:

  The r-direction:
  \[
  \rho \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_\theta \frac{\partial u_r}{\partial \theta} - \frac{u_z^2}{r} + u_z \frac{\partial u_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \rho g_r \\
  + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_r}{\partial r} \right) - \frac{u_r}{r^2} + \frac{2}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{u_z^2}{r^2} \frac{\partial^2 u_r}{\partial z^2} \right]
  \]

  The \( \theta \)-direction:
  \[
  \rho \left( \frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + u_\theta \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta \\
  + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_\theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_\theta}{\partial z} \right]
  \]

  The z-direction:
  \[
  \rho \left( \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_\theta \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \rho g_z \\
  + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial^2 u_z}{\partial z^2} \right]
  \]

- The Navier-Stokes (or momentum) equations in Cartesian coordinate are:

  \[
  \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho g_x - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\
  \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \rho g_y - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\
  \rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \rho g_z - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)
  \]

- Euler’s equations in streamlined coordinate:

  \[
  \frac{1}{\rho} \frac{\partial p}{\partial s} = -V \frac{\partial V}{\partial s}, \quad \frac{1}{\rho} \frac{\partial p}{\partial n} = \frac{V^2}{R} \\
  \tau_w = \frac{d}{dx} (U^2 \theta) + \delta^* \frac{dU}{dx}, \quad \delta^* = \int_0^\delta (1 - \frac{u}{U}) dy, \quad \theta^* = \int_0^\delta \frac{u}{U} dy \\
  \frac{\partial}{\partial t} \int_C \rho dV + \int_{CS} \rho V \cdot dA = 0, \quad \frac{\partial}{\partial t} \int_C \rho u dV + \int_{CS} \rho u V \cdot dA = F_{sx} + F_{bx}
  \]

- Table needed is shown below.
<table>
<thead>
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<th>Object</th>
<th>Diagram</th>
<th>$C_D(Re \geq 10^3)$</th>
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<td>Ring</td>
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