1. The controlled plant of a negative unity feedback system is shown in Figure 1 where

\[ G(s) = \frac{2}{s(s+1)(s+4)} \]

It is desired to compensate the system so as to meet the following transient response specifications:
- Damping ratio of dominant roots, \( \zeta \leq 0.5 \)
- Undamped natural frequency of dominant roots, \( \omega_n \leq 2 \text{ rad/sec} \)
- Velocity error constant, \( K_v \geq 6 \text{ sec}^{-1} \)

(a) Show the permissible area for the dominant closed-loop poles in order to achieve the desired response. (10%)

(b) Design a cascade lead compensator (choose the compensator zero so as to cancel the plant pole at \( s = -1 \)) so that the compensated system has a desired dominant closed-loop poles at \( s_d \) with \( \zeta = 0.5 \) and \( \omega_n = 2 \text{ rad/sec} \). (10%)

(c) Design a cascade lag compensator (choose the compensator zero at the intersection of the real axis and the line drawn from \( s_d \) making an angle of with the desired \( \zeta \)-line) so that the compensated system has \( K_v = 6 \text{ sec}^{-1} \). (10%)

![Figure 1](image)

2. Consider a feedback system shown in Figure 2 with

\[ G(s)H(s) = \frac{K(1+0.5s)(s+1)}{(1+10s)(s-1)} \quad ; \quad K > 0 \]

(a) Sketch the Nyquist plot of \( G(s)H(s) \). (10%)

(b) Use the Nyquist criterion to determine the range of values of \( K \) for which the system is stable. (10%)

![Figure 2](image)
3. The schematic diagram of a voice-coil motor (VCM), used as a linear actuator in a disk memory-storage system, is shown in Figure 3. The VCM consists of a cylindrical permanent magnet (PM) and a voice coil. When current is sent through the coil, the magnetic field of the PM interacts with the current-carrying conductor, causing the coil to move linearly. The voice coil of the VCM in Figure 3(a) consists of a primary coil and a shorted-turn coil. The later is installed for the purpose of effectively reducing the electric constant of the device. Figure 3(b) shows the equivalent circuit of the coils. The following parameters and variables are defined: $e_a(t)$ is the applied coil voltage, $i_a(t)$ the primary-coil current, $i_s(t)$ the shorted-turn coil current, $R_p$ the primary-coil resistance, $L_a$ the primary-coil inductance, $L_m$ the mutual inductance between the primary and shorted-turn coils, $v(t)$ the velocity of the voice coil, $y(t)$ the displacement of the voice coil, $f(t) = K_i i_a(t)$ the force of the voice coil, $K_i$ the force constant, $e_s(t) = K_b v(t)$ the back emf, $K_b$ the back-emf constant, $M$ the total mass of the voice coil and load, and $B$ the viscous-friction constant of the voice coil and load.

(a) (9%) Write the differential equations of the system.
(b) (10%) Draw a block diagram of the system with $E_a(s), I_a(s), I_s(s), V(s)$, and $Y(s)$ as variables.
(c) (6%) Derive the transfer function $G(s) = Y(s)/E_a(s)$.

![Figure 3](image)

4. Consider the system shown in Figure 4, where

$$D(s) = K \frac{s + \alpha}{s^2 + \omega^2}.$$  

(a) (10%) Find the steady-state error if the system is to track a sinusoidal reference input $r(t) = \sin \omega t$.
(b) (10%) Find the range of $K$ and $\alpha$ such that the closed-loop system remains stable.
(c) (5%) For the special case where $\alpha = 1$ is fixed, determine whether the closed-loop system would be asymptotically stable for some set of values for $K$.

![Figure 4](image)