1. (15%) **Axiom:** If \( \{C_n\} \) is a sequence of sets in a \( \sigma \)-field and \( C_i \) and \( C_j \) are disjoint for all \( i \neq j \), then we have \( P(\bigcup_{n=1}^{\infty} C_n) = \sum_{n=1}^{\infty} P(C_n) \). Now let \( \{A_n\} \) be a decreasing sequence of events. Then, applying this axiom, prove that

\[
\lim_{n \to \infty} P(A_n) = P(\lim_{n \to \infty} A_n) = P(\bigcap_{n=1}^{\infty} A_n).
\]

2. (12%) Let \( X \) and \( Y \) have the probability density function (pdf) \( f(x, y) = 8xy \) for \( 0 < x < y < 1 \); and \( f(x, y) = 0 \) elsewhere. If \( W = XY^2 \) and \( Z = X/Y \), then \( E(W) = \) and \( E(Z) = \)

3. (23%) Answer (or prove) the following questions:

1. What is the **Markov’s Inequality**? (5%)
2. What is the **Chebyshev’s Inequality**? (5%)
3. What is the **Jensen’s Inequality**? (5%)
4. Let \( X \) be a positive random variable.
   **Prove** that \( E(1/X) \geq 1/E(X) \). (8%)

4. (40%) Let \( X \) and \( Y \) have the joint pdf \( f(x, y) = 1 \), where \( |y| < x \) and \( 0 < x < 1 \).

1. Find the marginal pdf of \( X \). (2%)
2. Find the conditional expectation of \( Y \) given \( X = x \). (3%)
3. Find the marginal pdf of \( Y \). (5%)
4. Find the conditional expectation of \( X \) given \( Y = y \). (10%)
5. Evaluate the correlation coefficient of \( X \) and \( Y \). (10%)
6. Evaluate \( P(X + Y \leq 1) \). (10%)

5. (10%) Let \( X \) be from \( N(\mu, 1) \) and \( Y = [1 - \Phi(X)]/\phi(X) \), where \( \Phi \) and \( \phi \) denote the distribution function and the density of \( N(0, 1) \), respectively. Evaluate \( E[Y] \).