1. (10%) The NCHU Co. has determined that its variable costs are given by the following relationship:

\[ VC = 0.05Q^3 - 15Q^2 + 500Q \]

where Q is the quantity of output produced.

(a) Determine the output level where average variable costs are minimized.
(b) Determine the output level where marginal costs are minimized.

2. (15%) Taichung Oil’s fixed costs are $2,500,000 and its debt repayment requirements are $1,000,000. Selling price per barrel of oil is $20 and variable costs per barrel are $10.

(a) Determine the breakeven output (in dollars).
(b) Determine the number of barrels of oil that offshore must produce and sell in order to earn a target (operating) profit of $1,500,000.
(c) Determine the degree of operating leverage at an output of 400,000 barrels.

3. (15%) The Joy Company manufactures and sells a line of sewing machines. Demand per period (Q) for a particular model is given by the following relationship:

\[ Q = 400 - 0.5P \]

where P is price. Total costs (including a "normal" return to the owners) of producing Q units per period are:

\[ TC = 20,000 + 50Q + 3Q^2 \]

(a) Express total profits (\( \pi \)) in terms of Q.
(b) At what level of output are total profits maximized? What price will be charged? What are total profits at this output level?
(c) What model of market pricing has been assumed in this problem? Justify your answer.
4. (20%) Louisiana Gas Company is the sole producer of natural gas in the remote island country of Asia. The company's operations are regulated by the State Energy Commission. The demand function for gas in Asia has been estimated as:

\[ P = 1,100 - .2Q \]

where \( Q \) is output (measured in units) and \( P \) is price (measured in dollars per unit). Louisiana's cost function is:

\[ TC = 300,000 + 100Q \]

This total cost function does not include a "normal" return on the firm's invested capital of $4 million.

(a) In the absence of any government price regulation, determine Louisiana's optimal (i) output level, (ii) selling price, (iii) total profits, and (iv) rate of return on its asset base.

(b) The State Energy Commission has ordered the firm to charge a price which will provide it with no more than a 12 percent return on its total assets. Determine Louisiana's (i) output level, (ii) selling price, and (iii) total profits under this constraint.

5. (20%) Two companies (A and B) are duopolists that produce identical products. Demand for the products is given by the following demand function:

\[ P = 10,000 - Q_A - Q_B \]

where \( Q_A \) and \( Q_B \) are the quantities sold by the respective firms and \( P \) is the selling price.

Total cost functions for the two companies are:

\[ TC_A = 500,000 + 200Q_A + .5Q_A^2 \]
\[ TC_B = 200,000 + 400Q_B + Q_B^2 \]

Assume that the two firms act independently as in the Cournot model (that is, each firm assumes that the other firm's output will not change). Determine the long-run equilibrium output and selling price for each firm.
6. (20%) The ABC Tobacco Company feels that it is faced with the following segmented demand function for its cigarettes:

\[ P = \begin{cases} 
10 - 10Q & \text{When } 0 \leq Q \leq 20 \\
12 - 0.2Q & \text{When } Q > 20 
\end{cases} \]

where \( Q \) is the number of cartons sold and \( P \) is the price per carton.

(a) Why is such a segmented demand function likely to exist? What type of industry structure is indicated by this relationship?

(b) Determine ABC's marginal revenue function.

(c) Given that ABC's total cost function (including a "normal" return to the owners) is:

\[ TC_1 = 80 + 2.6Q + 0.05Q^2 \]

Determine ABC's profit maximizing price and output level.

(d) Given that ABC's total cost function increases to:

\[ TC_2 = 90 + 3.4Q + 0.05Q^2 \]

what is their profit maximizing price and output level?